# Aggregation with a double non-convex labor supply decision: indivisible private- and public-sector hours 

Aleksandar Vasilev*


#### Abstract

This paper explores the problem of non-convex labor supply decisions in an economy with both private and public sector jobs. To this end, Hansen (1985) and Rogerson's (1988) indivisible-hours framework is extended to an environment featuring a double discrete labor choice. The novelty of the study is that the micro-founded representation obtained from explicit aggregation over homogeneous individuals features different disutility of labor across the two sectors, which is in line with the observed difference in average wage rates (OECD 2011). Therefore, this theory-based utility function could be utilized to study labor supply responses over the business cycle.


Keywords: indivisible labor, public employment, aggregation, elasticity of labor supply
JEL Codes: J22, J30, J45
DOI: 10.17451/eko/47/2016/233

[^0]
## 1. Introduction

In the standard real business cycle model, as Cooley and Prescott (1995) have pointed out, changes in hours account for two-thirds of the cyclical output volatility. Those hours, however, are assumed to be supplied in the private sector only, and thus the private-public sector labor choice is ignored. This study adds to the literature by distinguishing between the two types of hours: after all, central governments in EU countries are the biggest employers at a national level, and public employment is a significant share of total employment. This paper goes one step further and focuses on the fact that workers work full-time and only very rarely move between public and/or private sector. Thus, the non-convexities (either work a full week on a job, or not work at all) in both sectors are taken seriously, and the study will try to uncover whether those features could produce interesting effects on the European labor markets. In particular, this "double indivisibility" of hours could provide new implications for the economy's behavior over the business cycle. Following Rogerson (1988) and Hansen (1985), this paper utilizes their approach by considering the effect of indivisibilities/non-convexities in both the private and public sector labor market. Using explicit aggregation, the resulting utility representation features constant, but different disutility of labor in the two sectors. This is an important finding, as it could easily accommodate the different wage rates observed in data. Such a micro-founded representation featuring a wedge between hours worked in the private vs. hours supplied in the public sector could be useful in future macroeconomic studies dealing with the propagation of business cycle fluctuations, as well as models dealing with fiscal policy effects working through the labor markets.

A side result of the aggregation procedure is to demonstrate that the representation used in Linnemann (2009), which is claimed to be derived using Hansen's (1985) indivisible hours setup, holds true only for a very special case. Hansen (1985), however, presents a model with one-sector, while Linnemann (2009) uses a model with two sectors: private and public. With two distinct labor markets, there are going to be two discrete decisions, so we need two different lotteries to convexify the two distinct hours decision sets. The implicit assumption used in Linnemann (2009) is that households decide on the sector of the economy in which to enter, and then conditional on the sector, decide whether to supply a fixed amount of hours, or none. Without proof, Linnemann (2009) claims that the resulting aggregate utility function is as below

$$
\begin{equation*}
U=\ln \left[C^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}-\Phi H \tag{1}
\end{equation*}
$$

where, $H=H^{p}+H^{g}$, and $C, S, H^{p}, H^{g}, H$ denote aggregate private consumption, government services, private hours, public hours and total hours, respectively.

In US data, the steady state hours for the two sectors are different, and that is the data Linnemann (2009) uses to calibrate his model. This creates an internal
inconsistency within his own model, and with a single wage rate in the setup, this also leads to indeterminacy of total hours. The indeterminacy is due to the fact that given the assumed common wage in the two sectors, and the equal disutility of work across sectors, additional information is needed to provide the split of hours between the two sectors. ${ }^{1}$ Linnemann (2009) solves this problem by assuming that public employment follows a stochastic process. We will show that given the calibration used in the original paper, the disutility of an hour work in the two sectors will not be equal in the general case. In addition, the setup in this paper will feature endogenous public sector supply of labor hours. Thus, in order to close the model wage rate in the two sectors need to be different, which is in line with the stylized facts in both the US and major EU economies.

## 2. Model Setup

The theoretical setup is a static economy without physical capital, where agents face a non-convex decision in a two-sector economy. ${ }^{2}$ Since the focus is on a oneperiod world, the model abstracts away from technological progress, population growth and uncertainty. There is a large number of identical one-member households, indexed by $i$ and distributed uniformly on the $[0,1]$ interval. The households will be assigned a sector "type", and after the type is revealed, each one decides whether to work in that sector or not. In the exposition below, we will use small case letters to denote individual variables and suppress the index $i$ to save on notation.

### 2.1. Households

Each household maximizes the following utility function

$$
\begin{equation*}
\operatorname{Max}_{\left\{c, h^{p}, h^{g}\right\}}\left\{\ln (\tilde{c})+\alpha \ln \left(1-h_{p}-h_{g}\right)\right\} \tag{2}
\end{equation*}
$$

Where:

$$
\begin{equation*}
\widetilde{c}=\left[\left(c^{p}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}} \tag{3}
\end{equation*}
$$

and $c, S, h^{p}, h^{g}$ denote aggregate private consumption, consumption of the public

[^1]good, hours worked in the private sector, and hours worked in the government sector. The parameter $\alpha>1$ measures the relative weight of leisure in the utility function. Total consumption $\tilde{c}$ is a Constant Elasticity of Substitution (CES) aggregation of private consumption and consumption of government services, where $\eta>0$ measures the degree of substitutablity between the two types of consumption. ${ }^{3}$

Each household is endowed with one unit of time that can be allocated to work in the private sector, work in the government sector, or leisure

$$
\begin{equation*}
h^{p}+h^{g}+l=1 \tag{4}
\end{equation*}
$$

The novelty is that the labor supply is discrete $h^{p} \in\left\{0, \bar{h}^{p}\right\}, h^{g} \in\left\{0, \bar{h}^{g}\right\}$, $0 \leq h^{p}+h^{g} \leq 1$. As in Gomes (2015), looking for a job will follow a "directed search" process: each household decides in each period whether to go to the public or to the provide sector (or, alternatively, is assigned a "sector type"). This process is stochastic and has two realization. The probability of going into the private sector (or being a "private-sector type") is

$$
\begin{equation*}
q=\frac{H^{p}}{H^{p}+H^{g}} \tag{5}
\end{equation*}
$$

where uppercase letter denote aggregate quantities, i.e. $H^{p}$ denotes aggregate hours in the private sector, and $H^{g}$ are the aggregate hours worked in the public sector. Then the probability of being a public sector type is

$$
\begin{equation*}
1-q=\frac{H^{g}}{H^{p}+H^{g}} \tag{6}
\end{equation*}
$$

This process is i.i.d. accros individuals, so the Law of Large Numbers holds: at the aggregate level q share of the households will be private sector type ( $h^{g}=0$ ), and $1-q$ share will be public sector type $\left(h^{p}=0\right)$. Once a particular sector is chosen, each household decodes on its labor supply. Note that the setup is quite general and allows for different wage rates per hour worked in the two sectors.

In addition to the work income households hold shares in the private firm and receive profit share $\pi$, with $\int_{0}^{1} \pi d i=\Pi .^{4}$ Income is subject to a lump-sum tax t , where $\int_{0}^{1} t d i=T$. Therefore, each household's budget constraint is

$$
\begin{equation*}
c^{j} \leq w^{j} h^{j}+\pi-t, j=p, g \tag{7}
\end{equation*}
$$

Households act competitively by taking the wage rates $\left\{w^{p}, w^{g}\right\}$, aggregate outcomes $\left\{C, S, H^{p}, H^{g}\right\}$ and lump-sum taxes $\{\mathrm{T}\}$ as given. Each household chooses $\left\{c^{j}, h^{p}, h^{g}\right\}$ to maximize (2) s.t. (3)-(7).

[^2]
## 3. Firms

Next, there is a single firm producing a homogeneous final consumption good, which uses labor as an only input. The production function is given by

$$
\begin{equation*}
Y=F\left(H^{p}\right), F^{\prime}>0, F^{\prime \prime}<0, F^{\prime}\left(\bar{H}^{p}\right)=0 \tag{8}
\end{equation*}
$$

where the last assumption is imposed to proxy capacity constraint.
The firm acts competitively by taking the hourly wage rate $\{w\}$, aggregate outcomes $\left\{C, S, H^{g}\right\}$ and policy variable $\{\mathrm{T}\}$ as given. Accordingly, $\left\{H^{p}\right\}$ is chosen to maximize static aggregate profit ${ }^{5}$

$$
\begin{equation*}
\max _{H^{p}} F\left(H^{p}\right)-w^{p} H^{p} \quad \text { st. } \quad H^{p} \geq 0 \tag{9}
\end{equation*}
$$

## 4. Government

There is also a government sector in this economy. The public authority hires employees to provide the public services. The technology of the public good provision use labor $H^{g}$ as an input, which is remunerated at a non-competitive wage rate $w^{g}=\gamma w^{p}$. Parameter $\gamma \geq 1$ will measure the fixed gross mark-up of government sector wage rate over the private sector one. ${ }^{6}$ Government production function is as follows:

$$
\begin{equation*}
S=S\left(H^{g}\right), S^{\prime}>0, S^{\prime \prime}<0, S^{\prime}\left(\bar{H}^{g}\right)=0 \tag{10}
\end{equation*}
$$

Where the last assumption guarantees that not all "public-sector types" will work in the production of the public good. ${ }^{7}$ In addition, the public good is a pure non-market output, thus it will not appear in the government budget constraint. The public sector wage bill is financed by levying a lump-sum tax $T$ on all households

$$
\begin{equation*}
w^{g} H^{g}=T \tag{11}
\end{equation*}
$$

In terms of fiscal instruments available at the government's disposal, the government takes total public sector hours, $H^{g}$, as given and sets the public sector wage rate, $w^{g}$, as a fixed mark-up above the competitive wage rate. In a sense, the government faces a supply curve for labor in the public sector and determines the demand for government employees. Lump-sum taxes will be residually chosen

[^3]to guarantee the budget is balanced.

## 5. Decentralized Competitive Equilibrium

Given the choice of $T$, a DCE is defined by allocations $\left\{c^{p}, c^{g}, h^{p}, h^{g}, S\right\}$, wage rates $\left\{w^{p}, w^{s}\right\}$, and firm's profit $\pi$ s.t. (i) all the households maximize utility; (ii) the private firm maximizes profit; (iii) the government budget constraint is balanced; (vi) all the markets are clear.

Characterizing the DCE: conditional on a sector, everyone doing the same working or not working - is not equilibrium.

Proof: Case (1): For any positive and finite wage, i.e. $0<w^{j}<\infty$, both sectors will want to hire a bit of labor. Hence, $h^{j}=0, j=p, g$ cannot be equilibrium because firm will have a positive labor demand for any finite wage, and households will have zero consumption, $c^{j}=0, j=p, q$, which is ruled out as an optimal choice from the monotonicity of the logarithmic utility. ${ }^{8}$

Case (2): $h^{i}=\overline{h^{j}}, j=p, q$ only if $w^{j}=0, j=p, q$ which follows form the assumption on both production technologies. At such wage rates both the firm and government will want to hire everyone, but no household will want to supply any labor. Thus having everyone working is not optimal either. $Q E D$

Hence, if there is a DCE, it must be that in equilibrium not everyone will get the same private consumption. Still, everyone consumes the same level of public goods, as it is assumed to be non-excludable and non-rivalrous. The households that work will have higher utility of private consumption, while those which do not work will enjoy more utility form leisure. Lastly, every household belonging to the same type will enjoy the same level of total utility.

Therefore, we will consider an equilibrium in which $\lambda_{p}$ of the people who go to the private sector, and $\lambda_{g}$ of the people who go to the public sector work $0<\lambda_{p}+\lambda_{g}<1$. Thus, $H^{p}=\lambda^{p} \overline{h^{p}}$, and $H^{g}=\lambda^{s} \overline{h^{g}}$.

From the firm's optimization problem we obtain the expression for the competitive hourly wage

$$
\begin{equation*}
F^{\prime}\left(\lambda^{p} \bar{h}^{p}\right)=w^{p} \tag{12}
\end{equation*}
$$

Hence, there will be positive economic profits amounting to

$$
\begin{equation*}
\pi=\Pi=F\left(\lambda^{p} \bar{h}^{p}\right)-F^{\prime}\left(\lambda^{p} \bar{h}^{p}\right) \lambda^{p} \bar{h}^{p}>0 \tag{13}
\end{equation*}
$$

which follow from the assumption that the production function features decreasing returns to scale. Next, equilibrium government output is

$$
\begin{equation*}
S=S\left(\lambda^{g} \bar{h}^{g}\right) \tag{14}
\end{equation*}
$$

[^4]and lump-sum tax revenue equals
\[

$$
\begin{equation*}
T=w^{g} \lambda^{g} \bar{h}^{g} \tag{15}
\end{equation*}
$$

\]

Now we will show the existence of a unique pair by analyzing a system of two non-linear equations. Those equations use the equality of utility of those who work and those who do not in the same sector. Households in the private sector are indifferent between working or not working:

$$
\begin{gather*}
\ln \left[\left(F^{\prime}\left(\lambda^{p} \bar{h}^{p}\right) \bar{h}^{p}+F\left(\lambda^{p} \bar{h}^{p}\right)-F^{\prime}\left(\lambda^{p} \bar{h}^{p}\right) \lambda^{p} \bar{h}^{p}-T\right)^{\eta}+\right. \\
\left.\left(S\left(\lambda^{g} \bar{h}^{g}\right)\right)^{\eta}\right]^{\frac{1}{\eta}}+\alpha \ln \left(1-\bar{h}^{p}\right)=  \tag{16}\\
\ln \left[\left(F\left(\lambda^{p} \bar{h}^{p}\right)-F^{\prime}\left(\lambda^{p} \bar{h}^{p}\right) \lambda^{p} \bar{h}^{p}-T\right)^{\eta}+\left(S\left(\lambda^{g} \bar{h}^{g}\right)\right)^{\eta}\right]^{\frac{1}{\eta}}+\alpha \ln (1)
\end{gather*}
$$

Similarly, households in the public sector are indifferent between working or not

$$
\begin{gather*}
\ln \left[\left(w^{g} \bar{h}^{g}+F\left(\lambda^{p} \bar{h}^{p}\right)-F^{\prime}\left(\lambda^{p} \bar{h}^{p}\right) \lambda^{p} \bar{h}^{p}-T\right)^{\eta}+\left(S\left(\lambda^{g} \bar{h}^{g}\right)\right)^{\eta}\right]^{\frac{1}{\eta}}+\alpha \ln \left(1-\bar{h}^{g}\right)= \\
\ln \left[\left(F\left(\lambda^{p} \bar{h}^{p}\right)-F^{\prime}\left(\lambda^{p} \bar{h}^{p}\right) \lambda^{p} \bar{h}^{p}-T\right)^{\eta}+\left(S\left(\lambda^{g} \bar{h}^{g}\right)\right)^{\eta}\right]^{\frac{1}{\eta}}+\alpha \ln (1) \tag{17}
\end{gather*}
$$

Substitute out the public sector wage rate with its equivalent expression form the government budget constraint

$$
\begin{equation*}
w^{g}\left(\lambda^{p}\right)=\gamma \omega^{p}=\gamma F^{\prime}\left(\lambda^{p} \bar{h}^{p}\right) \tag{18}
\end{equation*}
$$

Then do the same for the lump-sum taxes to obtain

$$
\begin{equation*}
T\left(\lambda^{g}, \lambda^{p}\right)=w^{g} \lambda^{g} \bar{h}^{g}=\gamma F^{\prime}\left(\lambda^{p} \bar{h}^{p}\right) \lambda^{g} \bar{h}^{g} \tag{19}
\end{equation*}
$$

Next, proving existence and uniqueness of optimal $\left(\lambda^{p}, \lambda^{g}\right) \in(0,1) \times(0,1)$ follows trivially form the Brower's Fixed Point and the assumptions on the functional forms of utility an d productivity functions. ${ }^{9}$

Also, observe that consumption of households in the private sector is not equal to those in the public sector due to idiosyncratic ("sector-type") shock in the beginning. Note that there are a lot of equilibria (in terms of the "names" of the people working), all of them with the same fraction of population $\lambda_{p}$ working in the private sector, and $\lambda_{g}$ working in the public sector.

Let $c_{w}^{j}, c_{n}^{j}, j=p, g$ denote the private consumption of individuals that work and those who do not, respectively, in each sector with $c_{w}^{j}>c_{n}^{j}, j=p, g$. Because of the presence of the public good and non-convexities, the First Welfare theorem does not hold, so this equilibrium is not PO. The Social Planner (SP) can

[^5]then improve upon the equilibrium by giving in each sector a consumption level independent of the fact whether they worked or not, $c_{w}^{j}=c_{n}^{j}, j=p, g$. We formally state this below.

Claim: The allocation $c_{w}^{j}, c_{n}^{j}, j=p, g$ is not efficient, i.e. there is an alternative allocation that a SP could choose that can make everyone better off.

Intuitively, the SP randomly chooses a fraction $\lambda_{p}, \lambda_{g}$ of individuals to work in eachsectorandgivesector-specific consumption $c^{j}=\lambda_{j} c_{w}^{j}+\left(1-\lambda_{j}\right) c_{n}^{j}, j=p, g$. We need that the bundle offered by the SP is feasible and makes everyone better off. ${ }^{10}$ Note that there is no perfect risk sharing (insurance) between sector types due to idiosyncratic shock in the beginning.

Proof: Showing feasibility is trivial because

$$
\begin{equation*}
q\left[\lambda_{p} c_{w}^{p}+\left(1-\lambda_{p}\right) c_{n}^{p}\right]=q \lambda_{p} c_{w}^{p}+q\left(1-\lambda_{p}\right) c_{n}^{p} \tag{20}
\end{equation*}
$$

Similarly

$$
\begin{equation*}
(1-q)\left[\lambda_{g} c_{w}^{g}+\left(1-\lambda_{g}\right) c_{n}^{g}\right]=(1-q) \lambda_{g} c_{w}^{g}+(1-q)\left(1-\lambda_{g}\right) c_{n}^{g} \tag{21}
\end{equation*}
$$

Next, it will be shown that the new allocation constitutes a Pareto improvement: SP is giving in expected value something better than the equilibrium allocation. Household in each sector are made better off.

[^6]\[

$$
\begin{gather*}
q\left\{\lambda_{p} \ln \left[\left(\lambda_{p} c_{w}^{p}+\left(1-\lambda_{p}\right) c_{n}^{p}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\lambda_{p} \alpha \ln \left(1-\bar{h}_{p}\right)+\right. \\
\left.\left(1-\lambda_{p}\right) \ln \left[\left(\lambda_{p} c_{w}^{p}+\left(1-\lambda_{p}\right) c_{n}^{p}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\left(1-\lambda_{p}\right) \alpha \ln (1)\right\}+ \\
(1-q)\left\{\lambda_{g} \ln \left[\left(\lambda_{g} c_{w}^{g}+\left(1-\lambda_{g}\right) c_{n}^{g}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\lambda_{g} \alpha \ln \left(1-\bar{h}_{g}\right)+\right. \\
\left.\left(1-\lambda_{g}\right) \ln \left[\left(\lambda_{g} c_{w}^{g}+\left(1-\lambda_{g}\right) c_{n}^{g}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\left(1-\lambda_{g}\right) \alpha \ln (1)\right\} \\
=q\left\{\ln \left[\left(\lambda_{p} c_{w}^{p}+\left(1-\lambda_{p}\right) c_{n}^{p}\right]^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\lambda_{p} \alpha \ln \left(1-\bar{h}_{p}\right)\right\}+ \\
(1-q)\left\{\ln \left[\left(\lambda_{g} c_{w}^{g}+\left(1-\lambda_{g}\right) c_{n}^{g}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\lambda_{g} \alpha \ln \left(1-\bar{h}_{p}\right)\right\}>  \tag{22}\\
q\left\{\lambda_{p} \ln \left[\left(c_{w}^{p}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\lambda_{p} \alpha \ln \left(1-\bar{h}_{p}\right)+\left(1-\lambda_{p}\right) \ln \left[\left(c_{n}^{p}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\right. \\
\left.\left(1-\lambda_{p}\right) \alpha \ln (1)\right\}+(1-q)\left\{\lambda_{g} \ln \left[\left(c_{w}^{g}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\lambda_{p} \alpha \ln \left(1-\bar{h}_{g}\right)+\right. \\
\left.\left(1-\lambda_{p}\right) \ln \left[\left(c_{n}^{g}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\left(1-\lambda_{g}\right) \alpha \ln (1)\right\}=q\left\{\lambda _ { p } \operatorname { l n } \left[\left(c_{w}^{p}\right)^{\eta}+\right.\right. \\
\left.S^{\eta}\right]^{\frac{1}{\eta}}+\lambda_{p} \alpha \ln \left(1-\bar{h}_{p}\right)+\left(1-\bar{h}_{p}\right)+\left(1-\lambda_{p}\right) \ln \left[\left(c_{n}^{p}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+(1- \\
q)\left\{\lambda_{g}\left[\left(c_{w}^{g}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+\lambda_{g} \alpha \ln \left(1-\bar{h}_{g}\right)+\left(1-\lambda_{p}\right) \ln \left[\left(c_{n}^{g}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}\right\}
\end{gather*}
$$
\]

Where the strict inequality in (22) follows form the concavity of the logarithmic function and the Constant Elasticity of Substitution (CES) aggregation of total consumption.

In conclusion, we can do better than the equilibrium allocation (we know this from the presence of the public good, but can improve on the distortion form the discrete labor supply decision) if we allow the Social Planner to randomize allocations, or offer employment lotteries. Thus, $H^{p}=\lambda_{p} \bar{h}_{p}$, then $\lambda_{p}=H^{p} / \bar{h}_{p}$. Similarly, $H^{g}=\lambda_{g} \bar{h}_{g}$, then $\lambda_{g}=H^{g} / \bar{h}_{g}$. Then, using that $C^{p}=q c^{p}$ and $C^{g}=(1-q) c^{g}$, aggregate utility function becomes

$$
\begin{gather*}
U=q \ln \left[\left(C^{p}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+q \lambda_{p} \alpha \ln \left(1-\bar{h}_{p}\right)+q\left(1-\lambda_{p}\right) \alpha \ln (1) \\
(1-q) \ln \left[\left(\lambda^{g}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}+(1-q) \lambda_{g} \alpha \ln \left(1-\bar{h}^{g}\right)+(1-q)\left(1-\lambda_{g}\right) \alpha \ln (1) \tag{23}
\end{gather*}
$$

Substitute out the expression for $\lambda_{p}, \lambda_{g}$ and drop the $\ln$ (1) terms to obtain

$$
\begin{gather*}
U=q \ln \left[\left(C^{p}\right)^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}} q \frac{\alpha \ln \left(1-\bar{h}_{p}\right)}{\bar{h}_{p}} H^{p}+(1-q) \ln \left[\left(C^{g}\right)^{\eta}+\right.  \tag{24}\\
\left.S^{\eta}\right]^{\frac{1}{\eta}}+(1-q) \frac{\alpha \ln \left(1-\bar{h}^{g}\right)}{\bar{h}_{g}} H^{g}
\end{gather*}
$$

Let $-\frac{q \alpha\left(1-\bar{h}^{p}\right)}{h_{p}}=\Phi_{1}$ and $-\frac{(1-q) \alpha \ln \left(1-\bar{h}^{g}\right)}{h_{g}}=\Phi_{2}$, where $\Phi_{1}, \Phi_{2}>0$ are constants. Then

$$
\begin{equation*}
U=\ln \left[C^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}-\Phi_{1} H^{p}-\Phi_{2} H^{g} \tag{25}
\end{equation*}
$$

Finally, assume that households are able to pool together their resources and equalize consumption across the two groups, i.e.: $C^{p}=C^{g}=C .{ }^{11}$ Alternatively, the government could achieve this through a suitable second round of lump-sum taxation. Note that such a redistribution provides a perfect insurance against the idiosyncratic "sector-type" shock in the beginning. ${ }^{12}$ We will thus obtain

$$
\begin{equation*}
U=\ln \left[C^{\eta}+S^{\eta}\right]^{\frac{1}{\eta}}-\Phi_{1} H^{p}-\Phi_{2} H^{g} \tag{26}
\end{equation*}
$$

As in Rogerson (1988) and Hansen (1985), the Frisch elasticity of labor supply changes (from one to infinity), but there is a different weight on disutility of leisure with respect to hours in each sector, i.e. $\Phi_{1} \neq \Phi_{2}$, as long as $\bar{h}^{p} \neq \bar{h}^{g}$, which in turn implies $\lambda_{p} \neq \lambda_{g},{ }^{13}$ hence $H^{p}=\lambda_{p} \bar{h}^{p} \neq \lambda_{p} \bar{h}^{p}=H^{g}$ which is the case in the data set for the US used by Linnemann (2009). He uses steady state public hours (normalized) to be 0.16 vs. 0.17 for the hours in the private sector. Note that only for the special case when $\bar{h}^{p}=\bar{h}^{g}=\bar{h}$, and $\lambda^{p}=\lambda^{g}$, which do not hold in data, Linnemann's (2009) representation, corresponding to a case when $\Phi_{1}=\Phi_{2}=\Phi$, will be correct. In a real-business-cycle model, when optimizing over public and private employment, and dividing side by side to the two optimality conditions, we can obtain

$$
\begin{equation*}
\frac{\Phi_{2}}{\Phi_{1}}=\frac{w^{g}}{w^{p}} \tag{27}
\end{equation*}
$$

In OECD (2011) data, the public sector wage features a significant premium, e.g. $\frac{w^{g}}{w^{p}}=1.2$ for Germany, hence $\Phi_{2}>\Phi_{1}$. This means that public sector workers have a higher disutility of labor, and need a higher reservation wage.

For the general case, which is supported by data (Gomes 2015), the disutility

[^7]of a marginal hour worked in the two sectors will be constant, but different. ${ }^{14}$ Such a setup can now easily accommodate different wage rates across sectors. In addition, the different weights on the labor supplies was generated endogenously and was driven by the different employment shares in the two sectors and the different work-weeks. ${ }^{15}$ This has important policy implications, as variations in total hours in data are due to variations in the number of people employed, and not due to variations in the hours worked per person. However, all this is left on the agenda for future research.

## 6. Conclusions

This paper explored the problem of non-convex labor supply decision in an economy with both private and public sector. To this end, Hansen (1985) and Rogerson's (1988) indivisible-hours framework was extended to an environment featuring a double discrete labor choice. The novelty of the study was that the microfounded representation obtained from explicit aggregation over homogeneous individuals features different disutility of labor across the two sectors, which is in line with the observed difference in average wage rates (OECD 2011). This theory-based utility function could be then utilized to study labor supply responses over the business cycle, and produce new implications of the economy's behavior over the business cycle.

## References

Cooley, Thomas F, and Edward C. Prescott, 1995. "Economic Growth and Business Cycles." In: Frontiers of Business Cycle Research, ed. Thomas F. Cooley, 1-38. Princeton: Princeton University Press.
Gomes, Pedro. 2015. "Optimal Public Sector Wages." The Economic Journal 125 (587): 1425-1451.

Hansen, Gary. 1985. "Indivisible Labor and the Business Cycle." Journal of Monetary Economics 16: 309-328.
Linnemann, Ludger. 2009. "Macroeconomic effects of shocks to public employment." Journal of Macroeconomics 31: 252-267.
OECD Statistical Databes. 2011. http://stats.oecd.org/(accessed: 17.10.2015). Rogerson, Richard. 1988. "Indivisible Lotteries, Lotteries and Equilibrium." Journal of Monetary Economics 21: 3-16.

[^8]
[^0]:    Asst. Professor and CERGE-EI Affiliate Fellow, Department of Economics, American University in Bulgaria, 1 Georgi Izmirliev Sq., Blagoevgrad 2700, Bulgaria, e-mail: avasilev@aubg.bg.

[^1]:    ${ }^{1}$ The optimality condition from the firm problem is not enough, we also need an optimality condition determining labor demand, i.e. a government choosing employment to balance the budget constraint.
    ${ }^{2}$ Adding physical capital accumulation decision and a dynamic structure to the model is then straightforward. Also, the absence of those elements in the current analysis does not affect in any major way the derivation of the optimality conditions characterizing the aggregate labor supply decisions.

[^2]:    ${ }^{3}$ The separability of consumption and leisure is not a crucial assumption for the results that follow. A more general, non-separable, utility representation, does not generate new results, while significantly complicates the algebraic derivations, and thus interferes with model tractability.
    ${ }^{4}$ This assumption guarantees a positive consumption to either of the two types, even if they choose not to work in their sector.

[^3]:    ${ }^{5}$ This representation can be viewed as being isomorphic to a problem in which capital has already been optimized over.
    ${ }^{6}$ Such a mark-up is stylized fact for the major EU economies.
    ${ }^{7}$ The level of government services increases households' utility, hence marginal utility matters.

[^4]:    ${ }^{8}$ If nobody works $\pi=0$ as well.

[^5]:    ${ }^{9}$ This theorem states that if a functions is continuous and monotone in its argument, and crosses the origin only once, than a unique fixed point exist on the domain over which the argument of the function is defined.

[^6]:    ${ }^{10}$ However, in the face of the uninsurable idiosyncratic shock, it is not the best improvement.

[^7]:    ${ }^{11}$ More precisely, the Social Planner can offer everyone from the two groups a consumption bundle $\hat{C}=q C^{p}+(1-q) C^{g}$. Showing feasibility is trivial, and the outcome that the new bundle will be strictly preferred follows form the concave shape of the utility function.
    ${ }^{12}$ Such a redistribution will only be efficient if performed after consumption levels have been equalized across states in both sectors. In particular, this redistribution is implemented once the lotteries and insurance markets have exhausted all possible profitable opportunities and have been closed.
    ${ }^{13} \lambda^{p}$ and $\lambda^{g}$ can be interpreted as sector-conditional job finding rates, as in Gomes's (2015) model with search and frictions. Also, in his calibration, $\lambda^{p} h \lambda^{g}$.

[^8]:    ${ }^{14}$ Extending our results to the infinite-horizon setting, i.e. the typical RBC model is straightforward. In a dynamic setting, and in infinite horizon setup, the Law of Large Numbers can be applied and everyone will work share of their time in the private sector, and $\lambda_{g}$ share of their time in the public sector every period.
    ${ }^{15}$ Generally, the working week in the public administration is shorter, i.e. $\bar{h}^{g}<\bar{h}^{p}$.

