Underestimated or overestimated: matching function elasticities biased due to worker inflows and outflows

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Abstract

Matching function elasticities are biased, that is, underestimated or overestimated, if inflows and outflows of job seekers are omitted. I analyse mathematically the direction of this bias and estimate parameters for random, stock-flow and job queuing models, adjusted econometrically for the temporal aggregation bias in the data. I examine various sets of endogenous and exogenous variables from registered unemployment data in Poland. Estimates show that unemployment affects matching more than discouragement, and that unemployment stock elasticity is underestimated if discouraged workers are neglected.

1. Introduction

I analyse how worker flows affect matching function elasticities for demand (job offers) and supply (job seekers). When worker flows, temporal or spatial data aggregation or search endogeneity are not considered, the ways in which demand and supply engage in a labour market matching process are misperceived. Here, I study the effects of on-the-job search and job-to-job moves, out-of-labour force search and discouraged workers. Employed job seekers may search for job-to-job moves. Their search behaviour determines the impact they exert on a matching process. Employed job seekers can command a higher reservation wage, and this affects their job acceptance rate (Petrongolo and Pissarides 2001). Moreover, employers may prefer employed job seekers, as their current employment can indicate their productivity level. Other effects concern worker inflows and outflows to and from inactivity. An out-of-labour-force search takes place if data inappropriately reflect unemployment (Petrongolo and Pissarides 2001). A person who moves from inactivity to employment is unemployed for some period, even if this is of short duration, however, the low frequency of data limits the identification of all such transitions in the labour market. The discouraged worker effect[1] refers, in turn, to groups of workers who flow in and out of the labour force when employment conditions change. They seek work when it is available, but discontinue the search process if unfavourable conditions prevail (Long 1953).

The impact of worker flows has been studied previously. Galuščák and Münich (2005) and Petrongolo and Pissarides (2001) analysed how matching function elasticities vary when we account for either on-the-job search or discouraged worker effects. Broersma and van Ours (1998) examined how to measure a pool of job seekers. In the current study I contribute to the literature and extend previous analyses by examining mathematically how matching function elasticities differ when various worker flows are simultaneously considered, including concurrent worker flows between activity and inactivity. I propose an alternative formula for the labour market tightness index (based on a vacancy inflow) and provide comparative results. I present findings for random, stock-flow and job queuing models. I estimate parameters of these models while adjusting econometrically for the temporal aggregation bias in the data (following Gregg and Petrongolo, 2005[2]). I analyse the Polish labour market, as its data properties allow for an assessment of the importance of the outflow from unemployment to inactivity.

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[1] The opposite of the discouraged worker effect is the added worker effect, which shows an increase in the labour force participation of secondary groups of workers in response to a job loss or wage cut of the main income earner in the household (Gong 2010).

[2] The solution of Coles and Petrongolo (2008) is intentionally omitted, as it does not provide the elasticities for the stocks directly.

Theoretical considerations prove that vacancy elasticities are often underestimated when worker flows are neglected. Unemployment elasticities are overestimated when we overlook the fact that unemployed individuals compete with other job seekers for job offers. The outcome of the discouraged worker effect depends on whether the unemployment pool primarily affects matching or discouragement. Details of this bias vary when we modify the definition of labour market tightness, but the overall direction of the bias does not. Estimated coefficients partially confirm these findings, the unemployment pool affects matching more than discouragement. Once we neglect discouraged workers, the unemployment stock elasticity is underestimated.

2. Effect of worker flows on elasticities: theory

The aggregate matching function can reflect random, stock-flow or job queuing models. A random model is based on stocks (Blanchard and Diamond 1994). A match takes place when a job seeker finds a job offer. In a stock-flow model (Coles and Smith 1998), new agents seek potential trading partners in stocks. If no trade occurs, they wait for new matching options. Thus, at equilibrium, a stock on one side of the market attempts to match an inflow of new trading partners from the other side of the market. A job queuing concept assumes large discrepancies between demand and supply. Job offers are distributed randomly, and job seekers have perfect information on trading opportunities, although a lack of vacancies means they wait for new job postings (Shapiro and Stiglitz 1984).

In each of the models above, the left-hand side of a matching function equation is either the total outflow from unemployment, the outflow from unemployment to employment or total hires. The right-hand side of an equation includes various measures of the job seekers and vacancies, and other variables (if we estimate the augmented matching function). Based on previous considerations of Galuščák and Münich (2005) and Petrongolo and Pissarides (2001), let us assume the following notation: X is the total outflow from unemployment, M is the outflow from unemployment to employment, D is the outflow from unemployment to the out-of-labour force, E is the employment, U is the unemployment and I is the inactivity. The Cobb-Douglas matching function for the stock-based model is:

$$M = AU^\alpha V^\beta \tag{1}$$

where α, β are the function elasticities on unemployment stock and vacancy stock respectively, and A is the efficiency parameter. It is assumed that $A = 1$.

If employed, unemployed and inactive job seekers apply for different vacancies, they are in different sections of the market. There is no bias, as long as we can distinguish vacancies dedicated to unemployed individuals. Congestion arises when we assume that different job seekers compete for the same job offers

(e.g., published on the Internet). Unemployed individuals comprise only a fraction of all the job seekers $\frac{U}{U+E+I}$ and the matching function for unemployed workers becomes:

$$M_U = \frac{U}{U+E+I}(U+E+I)^\alpha V^\beta \tag{2}$$

It is assumed that the numbers of employed and out-of-labour force job seekers change procyclically and that both flows depend on the labour market tightness index as follows:

$$E = \rho\left(\frac{V}{U}\right)^\varphi, \text{ where } \rho, \varphi > 0 \tag{3}$$

$$I = \mu\left(\frac{V}{U}\right)^\xi, \text{ where } \mu, \xi > 0 \tag{4}$$

The total outflow from unemployment is:

$$X = M_U + D, \text{ where } D = U^\gamma V^{-\eta} \tag{5}$$

D approximates the discouraged worker effect. If the outflow from unemployment to inactivity depends only on a cycle, then $\gamma = \eta$. If a period of unemployment affects the outflow, then $\gamma > \eta$ (Petrongolo and Pissarides 2001). We seek α and β, but the values we get from Equation (5), with (2), (3) and (4) are:

$$\frac{\partial X}{\partial U}\frac{U}{X} = \frac{M_U}{X}\left[\alpha + (1-\alpha)\frac{E(1+\varphi)+I(1+\xi)}{U+E+I}\right] + \gamma\frac{D}{X} \tag{6}$$

$$\frac{\partial X}{\partial V}\frac{V}{X} = \frac{M_U}{X}\left[\beta - (1-\alpha)\frac{\varphi E + \xi I}{U+E+I}\right] - \eta\frac{D}{X} \tag{7}$$

The expressions in square brackets show the bias that arises due to the job seekers' pool structure.[3] In Equation (7), if $\alpha < 1$, the vacancy stock elasticity is underestimated and the function elasticity on the unemployment stock is overestimated (Equation 6). The size of the bias is weighted by the "share" of the on-the-job seekers and out-of-labour-force seekers in the total pool of job seekers.

We can easily interpret the impact of the discouraged worker effect if we omit the inflows to the job seekers pool. The elasticities are:

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[3] This fraction extends to $\frac{E(1+\varphi)+I(1+\xi)}{U+E+I} = \frac{E+I+\varphi E+\xi I}{U+E+I}$, which is the share of E + I in the total pool of job seekers + $\frac{\varphi E+\xi I}{U+E+I}$. This is the "share" of employed and inactive job seekers, weighted by the elasticities of the labour market tightness indices, in the total pool of job seekers.

$$\frac{\partial X}{\partial U}\frac{U}{X} = \alpha - \frac{D}{X}(\alpha - \gamma) \tag{8}$$

$$\frac{\partial X}{\partial V}\frac{V}{X} = \beta - \frac{D}{X}(\beta + \eta) \tag{9}$$

The vacancy stock elasticity (Equation 9) is underestimated compared to the β value. The outcome for the unemployment stock elasticity (Equation 8) can be twofold. If $\alpha < \gamma$, which means that unemployment stock affects matching less than discouragement, the elasticity is overestimated. If $\alpha > \gamma$, which means that the unemployment stock affects matching more than discouragement, the elasticity is underestimated.

In the stock-flow model, the matching function is:

$$M = AU^{\alpha_1}V^{\alpha_2}u^{\beta_1}v^{\beta_2} \tag{10}$$

where u is the unemployment inflow, v is the vacancy inflow and α_1, α_2, β_1, β_2, are the function elasticities on the unemployment stock, vacancy stock, unemployment inflow and vacancy inflow, respectively. A is the efficiency parameter, it is assumed that $A = 1$.

Let us also assume that job seekers, other than unemployed individuals, compete for job vacancies with newly unemployed workers[4] (the unemployment inflow). Thus, the inflow constitutes a fraction of the pool of job seekers: $\frac{u}{U+E+I}$. A matching function for the unemployed individuals is:

$$M_U = \frac{u}{u + E + I}U^{\alpha_1}V^{\alpha_2}(u + E + I)^{\beta_1}v^{\beta_2} \tag{11}$$

while the total outflow from unemployment can be expressed as a sum of the matched pairs and discouraged workers (who are still recruited from the stock):

$$X = M_U + D \tag{12}$$

Substituting Equations (3), (4), (5) and (11) into (12) gives the elasticities:

$$\frac{\partial X}{\partial U}\frac{U}{X} = \frac{M_U}{X}\left[\alpha_1 + (1 - \beta_1)\frac{\varphi E + \xi I}{u + E + I}\right] + \gamma\frac{D}{X} \tag{13}$$

$$\frac{\partial X}{\partial V}\frac{V}{X} = \frac{M_U}{X}\left[\alpha_2 - (1 - \beta_1)\frac{\varphi E + \xi I}{u + E + I}\right] - \eta\frac{D}{X} \tag{14}$$

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[4] The hazard of finding a job is negatively related to the duration of unemployment, so newly unemployed workers are more competitive than other unemployed workers.

$$\frac{\partial X}{\partial u}\frac{u}{X} = \frac{M_U}{X}\left[\beta_1 + (1-\beta_1)\frac{E+I}{u+E+I}\right] \tag{15}$$

$$\frac{\partial X}{\partial v}\frac{v}{X} = \beta_2 \frac{M_U}{X} \tag{16}$$

while the unknowns are $\alpha_1, \alpha_2, \beta_1$ and β_2.

Overall bias, as in the random model, is weighted by the shares of matched and discouraged workers in the total outflow from unemployment. If we neglect various kinds of job seekers, the unemployment stock and unemployment inflow elasticities are overestimated (Equations 13 and 15). The vacancy stock elasticity (Equation 14) is underestimated, and that of the vacancy inflow (Equation 16) is unchanged but weighted by the share of the matches in the total outflow. Equations (15) and (16) reflect the assumption that the discouraged worker effect only affects the stocks. Its interpretation is analogous to that of the stock-based model (see Equations 8 and 9).

A separate analysis refers to the job queuing matching function:

$$M = AU^\alpha v^\beta \tag{17}$$

A is the efficiency parameter, it is assumed that *A* = 1. Let us assume that the formulae for on-the-job search, out-of-labour force search and discouraged worker effects are of the following forms:

$$E = \rho\left(\frac{v}{U}\right)^\varphi, \text{ where } \rho, \varphi > 0 \tag{18}$$

$$I = \mu\left(\frac{v}{U}\right)^\xi, \text{ where } \mu, \xi > 0 \tag{19}$$

$$X = M_U + D, \text{ where } D = U^\gamma v^{-\eta} \tag{20}$$

These are analogous to Equations (3), (4) and (5), but vacancy inflow is used here instead of vacancy stock. The particular elasticities are similar to Equations (6)–(9), but relate to the vacancy inflow.

We can extend the analysis for the stock-flow model if we modify the expressions for on-the-job and out-of-labour-force job seekers. Let us assume that these workers enter the market when new job offers appear. Their number depends on a vacancy inflow and not on a vacancy stock. Labour market data show that vacancies are of short mean duration and that the inflow of the job offers reflects new trading opportunities more accurately. These arguments justify our assumption.

$$E = \rho\left(\frac{v}{U}\right)^\varphi \tag{21}$$

$$I = \mu \left(\frac{v}{U}\right)^{\xi} \tag{22}$$

In this case, we get the following elasticities:

$$\frac{\partial X}{\partial U}\frac{U}{X} = \frac{M_U}{X}\left[\alpha_1 + (1 - \beta_1)\frac{\varphi E + \xi I}{u + E + I}\right] + \gamma\frac{D}{X} \tag{23}$$

$$\frac{\partial X}{\partial V}\frac{V}{X} = \alpha_2\frac{M_U}{X} - \eta\frac{D}{X} \tag{24}$$

$$\frac{\partial X}{\partial u}\frac{u}{X} = \frac{M_U}{X}\left[\beta_1 + (1 - \beta_1)\frac{E + I}{u + E + I}\right] \tag{25}$$

$$\frac{\partial X}{\partial v}\frac{v}{X} = \frac{M_U}{X}\left[\beta_2 - (1 - \beta_1)\frac{\varphi E + \xi I}{u + E + I}\right] \tag{26}$$

The unemployment inflow elasticity is overestimated, and discouragement affects only the stocks. The unemployment stock elasticity is overestimated if unemployment stock affects matching less than discouragement, and underestimated otherwise. The vacancy stock elasticity is underestimated due to the discouraged worker effect, but the structure of the pool of job seekers does not change this. This pool structure affects the vacancy inflow elasticity. It is underestimated as long as, $\beta_1 < 1$ but the size of the bias is weighted by the "shares" of the employed and inactive job seekers in the total pool of job seekers.

3. Effect of worker flows on elasticities: empirical findings

I used registered unemployment data to determine how the matching function elasticities vary when different sets of variables are used in the estimations. I used data from Poland, as these data allowed a comparison of two endogenous variables: the total outflow from unemployment, and the outflow from unemployment to employment. I used monthly seasonally adjusted data[5] (Appendix A compiles the main statistical properties of the time series) for the period January 1999–June 2013. During this time span, the unemployment and vacancy figures followed a loop around downward sloping Beveridge curve. Thus, the aggregate activity changes prevailed over reallocation processes (Blanchard and Diamond 1989). Labour market tightness indices indicated that during this period, on average, it was much harder to find a job than a worker.

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[5] Data refer to the job offers registered at public employment offices only. This number underestimates the true quantity of job vacancies, as few companies publish job advertisements in this way.

The matching function has been employed previously in Polish labour market data (see Roszkowska (2009) for a literature review). However, most of these analyses used the concept of augmented matching function and focused on the determinants of the efficiency of labour market matching. The exceptions are Gałecka-Burdziak (2016), who referred to temporal aggregation bias in the data, and Antczak *et al.* (2016) who referred to spatial aggregation bias in the data. Previous results did not preclude any of the models describing the labour market matching process, although the random model seemed to prevail (Gałecka-Burdziak 2016). I estimated parameters of the random, stock-flow and job queuing matching models. I dealt with the data temporal aggregation bias and adopted the Gregg and Petrongolo's solution (2005), which allowed me to compute particular elasticities. The overall number of matches is:

$$M_t = a_t U_t + b_t u_t + \varepsilon_{U_t} \tag{27}$$

where M_t is the number of unemployed individuals who found work during month t, U_t is the beginning-of-month t unemployment stock, u_t is the unemployment inflow during month t, a_t is the outflow rate from the unemployment stock: $a_t = 1 - e^{-\lambda_t}$, and b_t is the outflow rate from the unemployment inflow: $b_t = 1 - \frac{1-e^{-\lambda_t}}{\lambda_t}$. The probability of leaving the unemployment stock (λ_t) in the stock-based model is:

$$\lambda_t = e^{\alpha_0 + \alpha_1 ln\left(\frac{V_t}{U_t}\right)} \tag{28}$$

where V_t is the beginning-of-month t vacancy stock.
The stock-flow model assumes that the number of matches is:

$$M_t = U_t\left(1 - e^{-\lambda_t}\right) + u_t\left(1 - \frac{(1-p_t)\left(1-e^{-\lambda_t}\right)}{\lambda_t}\right) \tag{29}$$

where p is the instantaneous matching probability of the unemployment inflow and:

$$\lambda_t = e^{\alpha_0 + \alpha_2 ln\left(\frac{v_t}{U_t}\right)} \tag{30}$$

$$p_t = e^{\gamma_0 + \gamma_1 ln\left(\frac{V_t}{u_t}\right)} \tag{31}$$

where v_t is the vacancy inflow during month t.
The job queuing concept assumes random trade between unemployment stock and vacancy inflow only, and thus:

$$\lambda_t = e^{\alpha_0 + \alpha_1 ln\left(\frac{v_t}{U_t}\right)} \tag{32}$$

Table 1 presents the results. It displays the hazard rates, λ and p, and the elasticities. Model I presents the random matching model, model II refers to the stock-

flow trade mechanism, and model III shows the job queuing matching model. Total outflow from unemployment is the endogenous variable in models I, II and III. The outflow from unemployment to employment is the endogenous variable in models Ib, IIb and IIIb. Appendix B presents the specifications of the equations. The estimation method used was a nonlinear least squares and included the first order serial correlation in the disturbance term to deal with autocorrelation. ADF tests rejected the null hypothesis that a unit root existed in the residuals, so the models converged to the long-term equilibrium.

The results confirmed that each trading mechanism took place in the labour market. The unemployment stock elasticity was lower when the total outflow from unemployment was used instead of the outflow from unemployment to employment in the stock-based model (models I and Ib). The stock-flow model yielded the following results. The unemployment stock elasticity changed in the expected direction, although the unemployment inflow elasticity differed substantially. The vacancy stock elasticity changed in the opposite direction to that expected. The vacancy inflow elasticity experienced minor volatility, but the value differed in both estimates. In the job queuing model, the unemployment inflow elasticity did not change. The unemployment stock elasticity evolved in line with the mathematical implications, while vacancy inflow elasticity experienced changes inverse to those expected.

**Table 1. Estimates of matching function models, Poland 1999–2013**

Statistics	Equation					
	Model I	Model II	Model III	Model Ib	Model IIb	Model IIIb
α_1	0.3278*** (0.014)			0.2364*** (0.012)		
α_2		0.4069*** (0.044)	0.5304*** (0.025)		0.3504*** (0.040)	0.3882*** (0.019)
γ_1		0.3909*** (0.060)			0.4943** (0.243)	
λ	[0.0962]	[0.0610]	[0.0876]	[0.0412]	[0.0364]	[0.0377]
p	-	[0.3589]	-	-	[0.0508]	-
Elasticities on:						
Unemployment stock	[0.637]	[0.371]	[0.408]	[0.722]	[0.544]	[0.527]
Unemployment inflow	[0.047]	[0.243]	[0.043]	[0.047]	[0.099]	[0.043]
Vacancy stock	[0.311]	[0.136]	-	[0.245]	[0.057]	-
Vacancy inflow	-	[0.255]	[0.466]	-	[0.306]	[0.351]
R^2 adj. R^2	0.884 0.882	0.923 0.921	0.905 0.904	0.856 0.854	0.877 0.875	0.873 0.871

Statistics	Equation					
	Model I	Model II	Model III	Model Ib	Model IIb	Model IIIb
ADF test for residuals p-value	−16.06 0.00	−14.27 0.00	−15.38 0.00	−14.10 0.00	−13.23 0.00	−13.30 0.00
No. of obs.	174	174	174	174	174	174

Models I and Ib represent the random matching model, Models II and IIb represent the stock-flow matching model, Models III and IIIb represent the job queuing model. Models I, II and III: the dependent variable is the total outflow from unemployment. Model Ib, IIb and IIIb: the dependent variable is the outflow from unemployment to employment. Estimation method: nonlinear least squares. Each equation includes AR (1). Standard errors reported in brackets. * – significant at the 10% level, ** – significant at the 5% level, *** – significant at the 1% level. The matching elasticities λ and p are sample averages and are reported in square brackets.

Source: Author's own calculation.

Computation showed that elasticities varied over time and were interdependent in pairs that reflected certain trade mechanisms. For example, Figure 1 displays particular elasticities for a stock-flow model. Unemployment stock elasticity moved in parallel with that of the vacancy inflow, and the unemployment inflow elasticity moved in parallel with that of the vacancy stock. The unemployment stock elasticity was the highest during the analysed time span, apart from two exceptions. In the job queuing framework, when the endogenous variable was set to the total outflow from unemployment (model III), the vacancy inflow elasticity dominated. In the stock-flow model (model II), the unemployment inflow elasticity was largest around 2009.
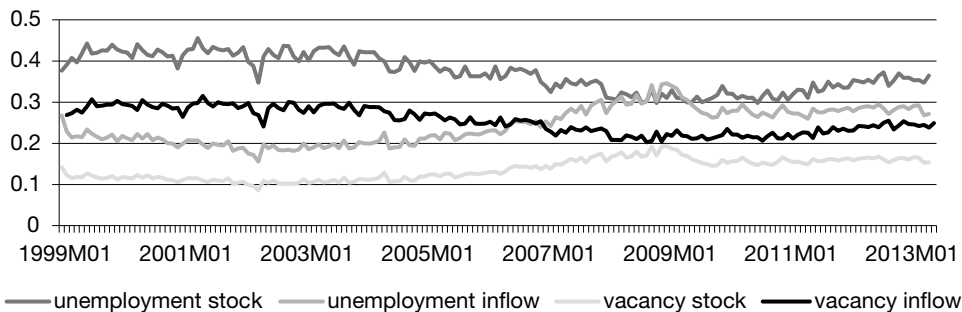
Figure 1. Matching function elasticities on the stock and inflow variables: stock-flow model (Eq. 2), Poland 1999–2013

Matching elasticities are computed on the basis of parameter estimates and data for each period.

Source: Author's own calculation.

During the analysed period, the mean exit rate from unemployment was 0.0974 ± 0.027. At the same time, the mean exit rate from unemployment to employment was 0.0425 ± 0.086. All the results underestimated these means. If the matching function included the vacancy inflow, the short-term volatility in the

exit rate was better reflected. More than a third of the newcomers left the unem-
ployment pool instantaneously, but this share decreased to 5% if the outflow from
unemployment to employment only was considered. The unemployment inflow
traded with the vacancy stock.

4. Discussion

Both the magnitude and direction of potential bias in the matching function elas-
ticities depend on certain assumptions. Employers can attract workers and direct
job offers to heterogeneous job seekers. These workers may be located within dif-
ferent sections of the market, so they do not compete for the same job offers. In
this case, no bias arises. However, when various job seekers apply for the same job
postings, congestion emerges. Thus, if we neglect worker flows into and out of the
labour force, the demand coefficient in the matching function is underestimated. If
the job seekers affect matching more than discouragement, the supply coefficient
suffers the same kind of bias. Moreover, if we modify the expressions for the num-
ber of on-the-job and out-of-labour-force job seekers, the findings change slightly,
but not in direction.

The left-hand-side variable of the matching function equation refers to the out-
flow from unemployment. Here, either the total outflow from unemployment or
the outflow from unemployment to employment is used. Thus, we can account for
externalities exerted by the worker flows on the unemployed job seekers. How-
ever, in this case, we do not consider matches generated by other job seekers. The
matches of employed or inactive workers are not reflected on the left-hand side of
the equation. We could derive separate expressions if the total number of hires was
used as the endogenous variable. Then, we could account for different kinds of em-
ployment inflows (from unemployment, employment or inactivity), but we could
not assess the impact of the outflow from unemployment to inactivity, that is, the
discouraged worker effect.

The empirical outcome partially supported the theoretical predictions. Esti-
mates confirmed the supply coefficient bias. This partial result may have origi-
nated in the method by which the data were collected. In Poland, the number of
matches reflects the worker flow. The outflow from unemployment to employment
does not necessarily originate only from public employment intermediation. It of-
ten exceeds the number of job offers available at employment offices, which are
underestimated. Thus, partially counterintuitive findings can arise from the proper-
ties of the data.

The quality of the data limits drawing further robust conclusions, although
some findings arise. The large difference in the value of p in the stock-flow stock-
flow model estimates (models II and IIb) suggests that there was a fraction of the
unemployed individuals who moved frequently in the market, while the rest re-

mained in the stock for longer periods. Thus, the unemployment stock turnover referred, in practice, to only a proportion of the stock. The data confirmed this hypothesis. The incidence of long-term unemployment was high and the majority of the unemployed workers exhibited low employability. Thus, only a few workers entered and exited the unemployment pool frequently.

5. Concluding remarks

In this study, I analysed theoretically and empirically the size of the bias which arises due to worker flows. When we choose variables, certain flows between employment, unemployment and inactivity are not generally considered. I extended previous analyses, and accounted simultaneously for job-to-job moves, out-of-labour-force searches, and the discouraged worker effect. I also showed how these findings change when the expressions for these worker flows are modified.

I verified theoretical conclusions empirically, and based the analysis on registered unemployment data for the Polish labour market. The computed elasticities did not confirm all the theoretical implications of the potential bias. The data showed that in Poland, the experience of unemployment did not demotivate workers, the unemployment pool affected matching more than discouragement. The unemployment stock elasticity changed as expected when various endogenous variables were used in the matching function estimations. Thus, the elasticities were underestimated when the outflow from unemployment to inactivity was neglected.

The research confirmed some of the theoretical implications. Nevertheless, it would be valuable to deepen this analysis and conduct estimations that produce more robust results. If the structures of the employment inflow and the pool of job seekers were accurately known, the size of the matching function elasticities bias could be assessed. Separate considerations of these could broaden the knowledge of the externalities which affect job offers when different recruitment methods are accounted for. Moreover, the vacancies are heterogeneous, as are the job seekers. I will address these issues in future research.

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Appendix A

Table A. Main statistical properties of variables, Poland, 1999–2013

	X	*M*	*U*	*u*	*V*	*v*
Mean	219781	97605	2391337	222164	33211	69839
Median	221445	95643	2263031	218356	35547	68262
Stand. Deviation	30871	13080	548703	19230	18930	19074
Monthly autocorr.	0.919	0.888	0.996	0.808	0.986	0.957
Min	153270	69448	1455690	188022	8503	34853
Max	283298	123948	3203591	276894	83356	112303
ADF	−1.78	−2.13	−1.60	−3.67	−1.12	−1.96
(p-value)	(0.39)	(0.23)	(0.48)	(0.00)	(0.71)	(0.30)
ADF[a]	−13.44	−12.83	−2.74	−18.27	−7.68	−15.15
(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
No. of obs.	174	174	174	174	174	174

[a] – calculated for first difference: *X* is the total outflow from unemployment, *M* is the outflow from unemployment to employment, *U* is the unemployment stock, *u* is the unemployment inflow, *V* is the vacancy stock, and *v* is the vacancy inflow.

Source: Registered unemployment 1999–2013, seasonally adjusted data, author's own calculation.

Appendix B

X_t is the total outflow from unemployment, M_t is the outflow from unemployment to employment, U_t is the beginning-of-month t unemployment stock, u_t is the unemployment inflow during month t, V_t is the beginning-of-month t vacancy stock, and v_t is the vacancy inflow during month t.

Models I and Ib:

$$O_t = \left[1 - e^{-e^{\alpha_0 + \alpha_1 ln\left(\frac{V_t}{U_t}\right)}}\right] U_t + \left[1 - \frac{1 - e^{-e^{\alpha_0 + \alpha_1 ln\left(\frac{V_t}{U_t}\right)}}}{e^{\alpha_0 + \alpha_1 ln\left(\frac{V_t}{U_t}\right)}}\right] u_t + \varepsilon_t$$

where $O_t = X_t$ in model I and $O_t = M_t$ in model Ib

Models II and IIb:

$$O_t = \left[1 - e^{-e^{\alpha_0 + \alpha_2 ln\left(\frac{v_t}{U_t}\right)}}\right] U_t + \left\{1 - \frac{\left[1 - e^{\gamma_0 + \gamma_1 ln\left(\frac{V_t}{u_t}\right)}\right]\left[1 - e^{-e^{\alpha_0 + \alpha_2 ln\left(\frac{v_t}{U_t}\right)}}\right]}{e^{\alpha_0 + \alpha_2 ln\left(\frac{v_t}{U_t}\right)}}\right\} u_t + \varepsilon_t$$

where $O_t = X_t$ in model II and $O_t = M_t$ in model IIb

Models III and IIIb:

$$O_t = \left[1 - e^{-e^{\alpha_0 + \alpha_2 ln\left(\frac{v_t}{U_t}\right)}}\right] U_{q-1} + \left\{1 - \frac{\left[1 - e^{-e^{\alpha_0 + \alpha_2 ln\left(\frac{v_t}{U_t}\right)}}\right]}{e^{\alpha_0 + \alpha_2 ln\left(\frac{v_t}{U_t}\right)}}\right\} u_t + \varepsilon_t$$

where $O_t = X_t$ in model III and $O_t = M_t$ in model IIIb